CALCULATED EFFECTS OF VARIOUS FACTORS ON THE FROST RESISTANCE OF CONCRETE

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The model of concrete as a "conglomerate within a conglomerate" is used to calculate the effects of various factors on the stresses in frozen concrete and its frost resistance.

The frost resistance of concrete is governed by the relative level of the tensile stresses which result from the differences in structure and thermal properties of the components of concrete; the problem of choosing a frost-resistant composition reduces to the minimization of the ratio σ_Σ/R_T (within technical and economic limitations) [1]. Below we use the concepts and results of [1] to analyze and carry out comparative calculations for the effects of various factors on the frost resistance of concrete. Before now, these effects could be determined only experimentally. It is now possible to explain on a common basis a variety of facts, many of which previously had no theoretical explanation or which required different explanations in different cases. The discussion below, like that in [1], is at the macroscopic level, i.e., within the framework of phenomenological considerations.

1. Effect of Hardening Time on the Frost Resistance of Concrete

It has been established experimentally that the frost resistance increases as a result of hardening, but no detailed explanation has been available. Analysis reveals that the frost resistance could increase as a result of three circumstances: a) a change in the ratio a_i/b_i as a result of the hydration of the Portland cement clinker, b) a shrinkage of the hydrated mass for the same reason, and c) an increase in the strength of the hydrated mass. Let us examine each of these factors quantitatively.

a) As a result of the hydration of the clinker particles, the average radius of the clinker relics decreases, and the concentration by volume of the hydrated mass increases. If the volume of the hydration products is, on the average, twice the volume of the clinker from which they are formed [2, 3], we can write the following equation on the basis of our calculation model for an element at structural level III:

$$v_{ei} = v_{1i} + 2(v_1^0 - v_{1i}), \tag{1}$$

where $v_1^0 = 1$. Simple manipulations yield

$$a_i = \sqrt[3]{v_{1i}}\, a_0 \quad \text{and} \quad b_i = \sqrt[3]{v_{ei}}\, b_0, \tag{2, 3}$$

where $a_0 = b_0$.

We write the degree of clinker hydration S as

$$S = 1 - \frac{v_{1i}}{v_1^0}. \tag{4}$$

Results calculated from Eqs. (1)-(4) are shown in Table 1; here S_i is the average experimental value. In calculating the stresses we used the data from [1] and assumed all the clinker particles to be of the same size. Since tangential cracks have a much greater effect than radial (contact) cracks on the strength of a structure, the data shown in Table 1 refer to tangential stresses.

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TABLE 1. Effect of the Time τ on the Stress σ'_{IIIt}

Characteristic	τ, days					
	0	14	28	180	365	1100
S_i	0	0,30	0,48	0,66	0,80	0,95
a_i/a_0	1,0	0,89	0,80	0,7	0,58	0,37
b_i/b_0	0	1,09	1,14	1,18	1,21	1,26
a_i/b_i	1,0	0,81	0,7	0,59	0,49	0,29
σ'_{IIIt}, kgf/(cm^2·deg)	0	14,0	13,1	11,2	9,8	8,2
$\sigma'_{IIIt}/\sigma'_{\tau=14}$	—	1,0	0,93	0,80	0,70	0,59

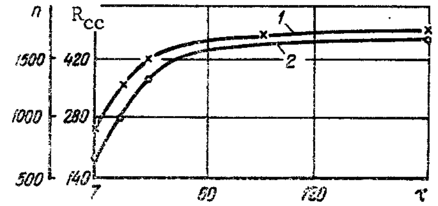

Fig. 1. Strength R_{cc} (kgf/cm^2) (1) and frost resistance n (2) of concrete as functions of τ (days).

We see from Table 1 that the stress σ'_{IIIt} falls off appreciably with increasing τ; after 3 yr (τ = 1100 days), e.g., its value is only 0.64 of that at τ = 28 days. In the example for which calculations were carried out in [1], the value $\sigma_{III} = 0.37\sigma_\Sigma$ was used for typical concrete. Accordingly, as a result of the decrease in $(a/b)_{III}$, the value of σ_Σ after 3 yr is only 0.36 × 0.37 = 0.13 of the value of σ_Σ at 28 days. We can use this value as a rough estimate.

b) Shrinkage of the hydrated mass reduces its water content and thus the relative strain upon freezing, because there is less of the expanding component in the former volume. After the initial hydration period (which we nominally set at τ_0 = 14 days), a rigid structure of hardened cement paste has formed, and the hardening products resulting from further hydration simply compress it. The degree of this compression can be estimated from Table 1; for the time interval in which we are interested we find $\Delta\rho_h = (1.26-1.14)/1.14 \approx 0.11$.

We assume a linear relation between $\Delta\rho_2$ and ε_2 during freezing over a sufficiently narrow range of densities and water contents. Since we have $|\varepsilon_2| \gg |\varepsilon_1|$ for the components at structural levels II and III during freezing, while we have $|\varepsilon_2| \sim |\varepsilon_1|$ for the component at structural level I, we can assume that there is also a linear relation between $\Delta\rho_2$ and $\Delta\varepsilon$. Accordingly, a change in ρ_h causes a proportionate change in σ_Σ.

c) Study of the effect of the hardening time on the properties of concrete has shown [4] that $(R_{ci}/R_{Ti})_\tau \approx$ const and $R_{c,\tau=1100}/R_{c,\tau=28} = 1.6 \div 2.0$.

For 3 yr concrete the decrease in σ_Σ is 0.13 + 0.11 = 0.24; thus the corresponding value of σ_Σ/R_T is lower than that for 28 day concrete by a factor which varies from 1.6/0.76 = 2.1 to 2.0/0.76 = 2.63. The experimental ratio of frost resistances for these two ages, also from [4], is from 1.93 to 2.5. Taking into account the nature of these estimates, we feel the agreement between the change in the criterion σ_Σ/R_T and that in the frost resistance is completely satisfactory. We should note here that the work in [4] was not carried out to establish the relationship between the frost resistance and the strength of concrete, but this relationship can be extracted from the data of [4], since we have 1.6/1.93 ≈ 2.0/2.5 within a small experimental error (the deviation from the mean is less than 2%).

It follows from this discussion that the increase in frost resistance is primarily a result of the strengthening of the concrete as a result of its hardening. However, it is difficult to make such a comparison at an appreciable number of points along the time axis on the basis of the data in the literature, since parallel studies of frost resistance and strength over various periods of time have been extremely rare. Figure 1 shows the behavior of the frost resistance and strength as functions of the hardening time. Curve 1 is taken from [3], while our curve 2 is plotted on the basis of an averaging of the results found in tests of five types of cement according to the ASTM classification [3]. This latter curve lies closest to the hardening curve for type I cement (ordinary Portland cement). Although the test results are not fully comparable, the similar behavior of the two curves can be taken as indirect evidence for a functional relationship between the strength and frost resistance. It therefore seems possible to calculate and predict the effect of the age of concrete on its frost resistance and even to estimate the effect of the density on the frost resistance.

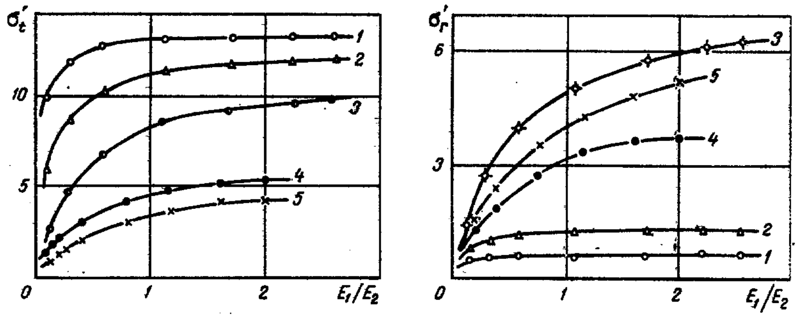

Fig. 2. Dependence of $\sigma'_{t,r,2}$ (kgf/cm^2·deg) on E_1/E_2: 1) $E_{2,II} = 1.8 \cdot 10^5$ kgf/cm^2, $a_{II} = 2.0$ mm; 2) the same, $a_{II} = 1.0$ mm; 3) the same, $a_{II} = 0.15$ mm; 4) $E_{2,I} = 2.5 \cdot 10^5$ kgf/cm^2, $a_I = 20$ mm; 5) the same, $a_I = 10$ mm.

2. Relationship between the Strength and Frost Resistance of Concrete

These data furnish a new basis for examining the old dispute over whether relatively strong concrete is also relatively frost resistant. The practical importance of this question is obvious. The problem has been that data on the effect of strength on frost resistance have been contradictory. Many samples of relatively strong concrete display a relatively poor frost resistance, while many other samples display a relatively good frost resistance.

Now, using the criterion σ_Σ/R_h, we can conclude that this question has not really been formulated correctly. The effect of strength on frost resistance is a bit complicated. The frost resistance, which is described by this fraction, depends not only on the strength (the denominator of the fraction) but also on the quantity σ_Σ (the numerator), and this numerator is in turn governed by a large number of far from obvious factors.

Nevertheless, we can assert at this point that, under otherwise equal conditions, relatively strong concrete is also relatively frost resistant.

3. Effect of the Elastic Modulus of the Aggregate on the Frost Resistance

It follows from the equations for the stresses σ_t and σ_r [1] that the stresses in the concrete are reduced as the elastic modulus of the inclusions is reduced. Figure 2 shows the stresses calculated for ordinary concrete with certain average values for the parameters [1]. An increase in the damping capability of the aggregate can significantly reduce σ_I and/or σ_{II}. If the aggregate is frost resistant, this stress decrease leads to a corresponding increase in the frost resistance of the concrete. For example, if the aggregate is a porous clay–sand mixture or gravel, for which the elastic moduli are usually far lower than for igneous rock, the concrete can be much more frost resistant than, for example, concrete made from crushed granite. This conclusion also follows from Fig. 2.

4. Effect of Composition on Frost Resistance

The composition, which is usually expressed in terms of the ratio of the components in the concrete mixture, can also be described in terms of various particular characteristics. Let us examine the sensitivity of the frost resistance to the ratio $(a/b)_{I,II}$, i.e., a parameter governed by the concentration by volume of the aggregate in the concrete conglomerate or by the amount of cement used. In our model the cement is described by $(b^3 - a^3)/b^3 = 1 - (a/b)^3_{I,II}$.

Figure 3 shows how the value of a/b affects the stresses in the model; for these calculations we used the elastic constants given in [1]. Figure 3 is plotted for a thawing half-cycle. If there is no hysteresis in the thermal strain, the freezing half-cycle simply changes the sign of the strain, without affecting its magnitude. The composition range of practical interest is from $a/b = 0.6$ to 0.9.

We see from Fig. 3 that an increase in the aggregate concentration (a decrease in the cement concentration) is accompanied by a significant increase in the stresses σ_{tI} and σ_{tII}, which would be associated

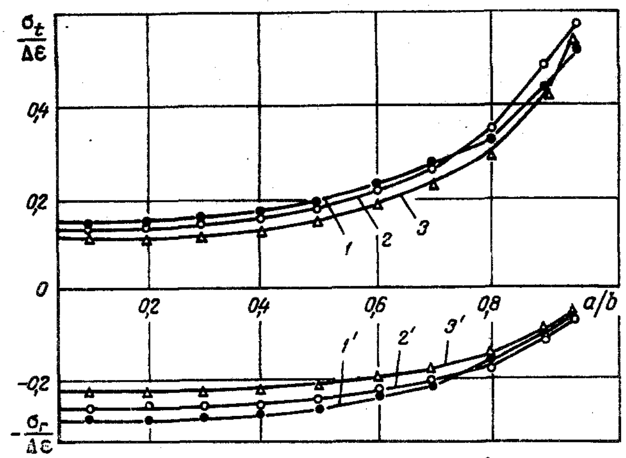

Fig. 3. Dependence of $\sigma_{t,r,2}/\Delta\varepsilon$ (kgf/cm^2) on the ratio a/b: 1, 1') structural level I; 2, 2') II; 3, 3') III. ($\sigma_{t,r}/\Delta\varepsilon \cdot 10^{-6}$).

with a reduction of the durability of the concrete [5]. A difference in the elastic constants of the components of heavy concrete has comparatively little effect on the stresses, at least the tangential stress: the curves remain essentially equidistant over a large range, up to $a/b \approx 0.75$. As the cement hardens and the clinker becomes hydrated, the ratio a/b decreases (Table 1), so that Fig. 3 also reflects the influence of this factor on the decrease in σ_{tIII} and the increase in the frost resistance of concrete as it hardens.

The influence of the ratio a/b on the frost resistance is actually more complicated than it would appear from Fig. 3, since the aggregate concentration also affects the value of ε_{con} at structural levels III and II; this effect can also be calculated [1]. Figure 3 is plotted for the case ε_{con} = const.

5. Effect of Pore-Forming Additives

One of the basic methods used to increase the frost resistance of concrete, in the Soviet Union and elsewhere, is to add pore-forming (air-entraining or gas-evolving) additives to the concrete mixture. Extensive experimental work has shown that when such additives are used the concrete displays no or a very low level of the anomalous expansion strain upon freezing which occurs in concrete without such additives [6-8]. The good frost resistance of concrete with such additives follows directly from the circumstance that the quantity $\Delta\varepsilon = (\alpha_1 - \alpha_2)\Delta t$ is very low in this case because α_1 and α_2 are approximately equal. Since the stresses at all structural levels are directly dependent on $\Delta\varepsilon$ [1], a decrease in this quantity sharply increases the frost resistance.

6. Effect of Freezing Temperature

The effect of the minimum temperature on the rate of damage is closely related to current problems of the testing of concrete for frost resistance and to the determination of the appropriate coefficients for converting the results of laboratory tests to applications in various types of construction.

According to [1], the stresses arise in the model in the interval $\Delta t = t_1 - t_{be}$, and the relative rate of damage or conversion coefficient k for concrete frozen to temperatures t_{i1} and t_{i2} is

$$k = \frac{t_{i2} - t_{be}}{t_{i1} - t_{be}}. \quad (5)$$

For example, with $t_{be} = -10°C$, $t_{i1} = -18°C$, and $t_{i2} = -45°C$, we would have $k \approx 4$ in agreement with the experimental data of [9].

7. Freezing Not Involving Passage through 0°C

It has been established that concrete suffers damage during temperature fluctuations below 0°C, even if the temperature does not rise to this point [9]. The theoretical explanation for this experimental fact is obvious: $t_{be} < 0°$. Equation (5) can be used for a rough comparison of the relative damage rates for concrete subjected to cyclic freezing and thawing, with or without a transition through 0°. We note that a complete thawing results in a significant redistribution of the water, whose distribution in the structure strongly influences the rate of damage during cyclic freezing [8].

8. Analysis of the Characteristic Types of Damage

Many experiments have demonstrated that there are basically two types of damage. First, the concrete conglomerate may be broken into several parts by a few large cracks; in this case the resulting parts retain a comparatively high strength. Second, there may be a general, quite uniform disintegration of the structure; in this case the concrete loses strength throughout its volume ("crumbles"). Mixed types of destruction have also been observed. Which type of destruction actually occurs has been treated as unpredictable, and in most cases the investigators have simply reported the type of destruction as an experimental fact.

Let us assume that macroscopic cracks appear at structural level I, while the ruptures which occur at levels II and III are "microscopic" cracks. Then the probability for the formation of macroscopic cracks can be written

$$P(M) = \frac{\sigma_{I}}{\sigma_{\Sigma}}, \tag{6}$$

while the difference between this probability and 1,

$$P(m) = \frac{\sigma_{II} + \sigma_{III}}{\sigma_{\Sigma}} \tag{7}$$

represents the probability for the formation of microscopic cracks.

According to the structural model of [1], the size of the macroscopic cracks is comparable to the linear dimension of the large aggregate in the concrete (the crushed stone or gravel), and several such cracks, by combining, cause the type of destruction in which the concrete breaks into several, comparatively large pieces. This type of destruction occurs if $P(M) > P(m)$ and is more clearly expressed, the stronger this inequality. The second type of destruction is observed if $P(m) > P(M)$; with $P(M) \approx P(m)$ the destruction is evidently of a mixed nature.

9. Frost Resistance of Cement Materials at Various Structural Levels

The linear dimensions of the aggregate particles, including the clinker grains, range from 100 mm to 10 μ, i.e., over four orders of magnitude. The components of concrete which we are examining at various structural levels, on the basis of the particle size [1], are used in practice as individual materials. For example, a cement–sand mixture is widely used structurally in fine-particle concrete and in reinforced concrete. Experience in construction and many experiments have established the good frost resistance of fine-particle concrete, which is up to 1.5-3 times better than that of concrete in which the cement–sand mixture has analogous characteristics. In turn, hardened cement paste is even more frost resistant.

The theoretical explanation for these facts follows from the concept of the combination of stresses in the model of a "conglomerate within a conglomerate," since the stress σ_{Σ} is lower, the lower the structural level under otherwise equal conditions. In particular, the increase in the frost resistance of fine-particle concrete over that of concrete with a large-particle aggregate should be proportional to $\sigma_{\Sigma}/(\sigma_{II} + \sigma_{III})$; for the example used in calculations in [1], we find the average value $1/0.72 \approx 1.4$, in agreement with the experimental results given above.

A similar explanation can be found for the decrease in the frost resistance of concrete made from an aggregate in which the distribution of particle sizes is not continuous [3]: the absence of certain particle sizes can produce new structural levels in the concrete and thereby affect its frost resistance.

10. Effect of a Combination of External Facts on the Frost Resistance

The actual structures which are subjected to freezing are in a loaded, stressed state, while the standard tests for frost resistance are usually carried out with unloaded samples. Accordingly, special studies have been carried out to determine how the frost resistance is affected by the particular nature and magnitude of the stresses caused by external forces [10-12].

Using the concept of frost resistance as the resistance of the material to structural stresses during freezing and thawing [1], we find a natural explanation for these results.

External loads, the thermoelastic stresses due to temperature gradients, the stresses which arise in reinforced concrete during freezing because of the difference between the thermal properties of steel and concrete [7-10, 13], etc., produce a stress field which is superimposed on the "inherent" structural stresses in concrete. Assuming in a first approximation that the superposition principle is valid here, we can write the strength criterion for the frost resistance in the case of a combination of external facts as

$$\frac{\Sigma\sigma}{R_{\text{T}}} < 1, \tag{8}$$

where

$$\Sigma\sigma = \sigma_{\Sigma} + \sigma_{ex} + \sigma_{st} + \sigma_{te}.$$

Depending on the relative importance of each of these contributions to the total stress $\Sigma\sigma$ arising in the concrete, various types of corrosion damage may also be observed during freezing. In particular, this situation is found in a particular type of damage which occurs at the ends of prestressed and loaded reinforced-concrete girders [9, 13]. It is pertinent to note that data on the effect of fixed external loads on the frost resistance of concrete can be used to determine the actual stresses arising in concrete during freezing and to convert from a discussion of a model to the actual material. We will not pursue these points further in this paper.

The number of factors affecting the durability of concrete is extremely large; an ASTM committee on the durability of concrete distinguished 190 such factors [14]. But even if we consider only 24 factors, the number of different combinations in which they could affect frost resistance is extremely large; a calculation shows that this number is of the order of 10^{30}. Accordingly, there is little hope for a purely experimental approach to the study of frost resistance, and theoretical work is necessary. This paper should be considered an effort in this direction.

This entire analysis has been based on the model of concrete as a "conglomerate within a conglomerate," in which various structural levels are distinguished. On the whole, this model leads to a description of the behavior of concrete upon freezing which agrees with experiment, so that this model will presumably be useful for studying sulfate attack, shrinkage, swelling, and other effects associated with stress and strain in a concrete structure.

NOTATION

σ, σ', and $\Sigma\sigma$	are the stresses in the envelope of the model, specific stresses, and integral stresses;
E	is the elastic modulus;
R	is the (local) strength;
v	is the concentration by volume;
a	is the core radius in the model;
b	is the outer radius of the envelope;
S	is the degree of hydration;
τ	is the time;
ε	is the strain;
$\Delta\varepsilon = \varepsilon_1 - \varepsilon_2$;	
ρ	is the density;
$\Delta\rho$	is the change in density;
α	is the thermal-expansion coefficient;
t	is the temperature.

Subscripts

Σ	denotes total;
T	denotes tensile;
c	denotes compressional;
I, II and III	denote structural level;
0	denotes initial;
i	denotes instantaneous;

In the model:

e denotes the element;
1 denotes the core;

2 denotes the envelope;
τ denotes the time (days);
h denotes the hydrated mass;
t denotes tangential;
r denotes radial;
ex denotes the external forces;
te denotes thermoelastic;
st denotes steel;
be denotes the temperature of the beginning of expansion;
con denotes conglomerate.

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